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Automorphisms as brane non-local transformations

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The relation among spacetime supersymmetry algebras and superbrane actions is further explored. It is proved that $SL(2,\mathbb{R})$ belongs to the automorphism group of the $\mathcal{N}=2$ $D=10$ type IIB SuperPoincaré algebra. Its $SO(2)$ subgroup is identified with a non-local $SO(2)$ transformation found in hep-th/9806161. Performing T-duality, new non-local transformations are found in type IIA relating, among others, BIon configurations with two D2-branes intersecting at a point. Its M-theory origin is explained. These results show that part of the SuperPoincaré algebra automorphism group might be realized on the field theory as non-local transformations.

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I. INTRODUCTION

The relation among spacetime supersymmetry algebras and superbrane effective actions is nowadays quite well understood [1]. The latter provide a field theory realization of the former. It then follows that there must exist, to certain extent, a parallelism among a pure algebraic approach to M-theory (string theory) and a field theory approach on branes. In particular, BPS states can be algebraically characterized by the saturation of a BPS bound associated with the positivity of the matrix $<\alpha|\{Q_\alpha\,,Q_\beta\}|\alpha>\geq 0$ for all Clifford valued states $|\alpha>$ [2]. Such states do have a field theory description in terms of a special class of field configurations, the so called *BPS configurations*. These saturate a bound on the field theoretic energy functional whenever certain functional constraints or *BPS equations*, are satisfied [3]. The latter can be derived [4] either from a direct hamiltonian analysis or from the resolution of the kappa symmetry preserving condition ($\Gamma_\kappa\epsilon=\epsilon$) [5] that any supersymmetric (bosonic) configuration must satisfy.

It has been lately stressed [6–9] that the maximal automorphism group of the $\mathcal{N}=1$ $D=11$ SuperPoincaré algebra is $GL(32,\mathbb{R})$. This raises a natural question : is there any field theory realization for such automorphism group or a subgroup of it ?. In [6], it was already pointed out the existence of an M5-brane symmetry corresponding to one of such automorphism transformations. One of the purposes of this paper is to go in this direction.

We shall concentrate on branes propagating in SuperPoincaré superspace. Since the Lorentz group is a subgroup of the corresponding automorphism group, it is already clear that such a subgroup will be linearly realized on the brane (before any gauge fixing). On the other hand, since "central charges" appearing in maximal SuperPoincaré algebras admit a field theory realization in terms of topological charges given by world space integrals involving derivatives of the brane dynamical fields, and they are generically "rotated" under automorphism transformations [9], one should also expect, if any, the existence of non-local transformations in the field theory side.

We shall provide evidence for the existence of such non-local transformations. We shall start by analysing the automorphism group of the $\mathcal{N}=2$ $D=10$ type IIB SuperPoincaré algebra. It will be shown that such group contains an $SL(2,\mathbb{R})$ factor, the corresponding U-duality group in ten dimensional type IIB theory [1]. We shall identify its $SO(2)$ subgroup with the non-local $SO(2)$ transformations found in [10] by analyzing them on some particular class of on-shell BPS configurations, dyons [3]. Performing a longitudinal T-duality transformation, we shall find new non-local transformations leaving the D2-brane effective action invariant. The new feature of the latter transformations is that they also involve bosonic scalar matter fields. The existence of these transformations again matches the corresponding $SO(32)$ automor-

[1] It would be interesting to know whether this relation among the automorphism group and the U-duality group extends to lower dimensional superalgebras.

phism transformations of the $\mathcal{N}=2$ $D=10$ type IIA SuperPoincaré algebra derived from a pure algebraic formulation describing T-duality among type IIA and type IIB superalgebras [11]. When applied on-shell, they map (among others) fundamental strings ending on the D2-brane (BIons) with two intersecting D2-branes at a point ($D2 \perp D2(0)$). We end up with the M-theory interpretation of these new type IIA transformations. These are rotations (elements of $SO(10) \in SO(32)$) involving the extra (eleventh) dimension. This interpretation is consistent with the well-known fact that an S-duality transformation in type IIB is a rotation interchanging the two independent cycles on the 2-torus in M-theory [12]. From the field theory perspective (M2-brane effective action), the origin of the type IIA non-local transformations is the three dimensional world volume dualization [13], needed to map the membrane action to the D2-brane action, which transforms the linearly realized rotations into the forementioned non-local ones. In this way, consistency of these new transformations with the known web of dualities in M/string theories is proved.

II. S-DUALITY, AUTOMORPHISMS AND D3-BRANES

The basic anticommutation relation defining the type IIB $\mathcal{N}=2$ $D=10$ SuperPoincaré algebra [2] is given by

$$\{Q^i,Q^j\} = \mathcal{P}^+\Gamma^M Y^{ij}_M + \mathcal{P}^+\frac{1}{3!}\Gamma^{MNP}\epsilon^{ij}Y_{MNP} + \mathcal{P}^+\frac{1}{5!}\Gamma^{M_1\ldots M_5}Y^{+ij}_{M_1\ldots M_5}\,, \tag{1}$$

where

$$Y^{ij}_M = \delta^{ij}Y^{(0)}_M + \tau_1^{ij}Y^{(1)}_M + \tau_3^{ij}Y^{(3)}_M \tag{2}$$

$$Y^{+ij}_{M_1\ldots M_5} = \delta^{ij}Y^{+(0)}_{M_1\ldots M_5} + \tau_1^{ij}Y^{+(1)}_{M_1\ldots M_5} + \tau_3^{ij}Y^{+(3)}_{M_1\ldots M_5}\,. \tag{3}$$

It would be important in the following to remember that all previous charges appearing in the right hand side of equation (1) are associated with single $\nu=\frac{1}{2}$ BPS branes in certain directions of spacetime. In particular, the three form spacelike components Y_{mnp} describe D3-branes standing along the mnp-hyperplane. Analogously, the one form spacelike components $Y^{(1)}_m$ and $Y^{(3)}_m$ correspond to D-strings and fundamental strings stretching along the x^m direction, respectively, whereas the five form spacelike components $Y^{+(1)}_{m_1\ldots m_5}$, $Y^{+(3)}_{m_1\ldots m_5}$ and $Y^{+(0)}_{m_1\ldots m_5}$ describe D5-branes, NS5-branes and KK5B monopoles along the $m_1\ldots m_5$-hyperplane, respectively.

[2] We are using the same notation and coventions as those used in [11].

Just as for the $\mathcal{N}=1$ $D=11$ SuperPoincaré algebra, one could ask about its maximal automorphism group. The latter certainly contains an $SL(2,\mathbb{R})$ factor acting on the internal indeces i,j. If we consider the transformation $\tilde{Q}^i=(U\,Q)^i$, $U\in \mathrm{SL}(2,\mathbb{R})$, the latter will indeed be an automorphism of the algebra (1) if the charges on the right hand side transform as

$$\tilde{\mathcal{Z}}^{ij} = \left(U\mathcal{Z}U^t\right)^{ij}\,. \tag{4}$$

Let us briefly study the elementary transformations generated by $U_a=e^{\alpha\tau_a/2}$ $a=1,3$ and $U_2=e^{\alpha i\tau_2/2}$, where $\{\tau_A \quad A=1,2,3\}$ is the set of Pauli matrices. Direct application of the transformation law (4) determines the corresponding charge transformations. Since $U_a=U_a^t$, it can be checked that $Y^{(0)}_M$ and $Y^{(a)}_M$ transform under an $SO(1,1)$ transformation

$$\begin{pmatrix}\tilde{Y}^{(0)}_M\\ \tilde{Y}^{(a)}_M\end{pmatrix} = S\begin{pmatrix}Y^{(0)}_M\\ Y^{(a)}_M\end{pmatrix} \tag{5}$$

where

$$S=\begin{pmatrix}\cosh\alpha & \sinh\alpha\\ \sinh\alpha & \cosh\alpha\end{pmatrix}\in \mathrm{SO}(1,1)\,. \tag{6}$$

There exists an analogous tranformation for the doublet $\left(Y^{+(0)}_{M_1\ldots M_5},\,Y^{+(a)}_{M_1\ldots M_5}\right)$, all other charges remaining invariant, due to the Pauli matrices algebra. Notice that $Y^{(0)}_0$ transforms, so that the energy is not left invariant under such transformations.

On the other hand, since $U_2^{-1}=U_2^t$, the subgroup generated by $U_2(\alpha)$ transformations corresponds to the $SO(2)$ subgroup preserving the energy. In this case $Y^{(1)}_M$ and $Y^{(3)}_M$ transform as a doublet under the $SO(2)$ transformation

$$\begin{pmatrix}\tilde{Y}^{(1)}_M\\ \tilde{Y}^{(3)}_M\end{pmatrix} = R\begin{pmatrix}Y^{(1)}_M\\ Y^{(3)}_M\end{pmatrix} \tag{7}$$

where

$$R=\begin{pmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{pmatrix}\in \mathrm{SO}(2)\,. \tag{8}$$

Analogous transformation properties are shared by the five form pair $\{Y^{+(1)}_{M_1\ldots M_5},\,Y^{+(3)}_{M_1\ldots M_5}\}$. All other charges remain invariant, including Y_{mnp} and $Y^{+(0)}_{m_1\ldots m_5}$.

Notice that $U_2(\alpha)$ includes S-duality transformations. Indeed, $U_2(\pi/2)$ interchanges D-strings with fundamental strings, and D5-branes with NS5-branes, whereas D3-branes and KK5B monopoles are left invariant, in agreement with their known S-selfdual properties.

We shall concentrate in the following on $U_2(\alpha)$ transformations and their world volume realization. The natural brane effective action where one might look for such

transformations is the D3-brane effective action. There are two basic reasons for such choice. The first one is the breaking of the automorphism group by the presence of the brane [7]. Due to the S-duality covariance of the D3-brane effective action, this seems to be a good choice. Furthermore, such classical action is known to admit solitonic solutions corresponding to (p,q)-strings (dyons). It is then natural to look for symmetry transformations relating (p,q)-strings with (p',q')-strings.

We shall thus concentrate on D3-branes propagating in SuperPoincaré. Its $\mathcal{N}=2$ supersymmetry and kappa invariant action is given by

$$\begin{aligned} S_{D3} &= \int d^4\sigma \ (\mathcal{L}_{DBI} \ + \ \mathcal{L}_{WZ}) \\ \mathcal{L}_{DBI} &= - \sqrt{-\det(\mathcal{G}_{\mu\nu} + \mathcal{F}_{\mu\nu})}\,, \\ \mathcal{L}_{WZ} &= C_{(2)}\ \mathcal{F} \ + \ C_{(4)}\,, \end{aligned} \tag{9}$$

where

$$\begin{aligned} \mathcal{G}_{\mu\nu} &= \Pi^m_\mu \Pi^n_\nu \eta_{mn} \\ \mathcal{F} &= d\,V + \Omega_3\,, \end{aligned} \tag{10}$$

are defined in terms of the supersymmetric invariant one form

$$\Pi^m = dx^m + \bar{\theta}\Gamma^m d\theta\,, \tag{11}$$

whose components are $\Pi^m_\nu = \partial_\mu x^m - \bar{\theta}\Gamma^m\partial_\mu\theta$ [3], and

$$\Omega_i = \bar{\theta}\hat{\not{\Pi}}\tau_i d\theta \tag{12}$$

where $\hat{\not{\Pi}} = \hat{\Pi}^m\Gamma_m$ and $\hat{\Pi}^m = \Pi^m - \frac{1}{2}\bar{\theta}\Gamma^m d\theta$. The RR-fields can be found in [15]. It will be important for us just to know that $C_{(2)} = \Omega_1$.

The transformations we are interested in were actually found in [10]. In that paper, the action (9) was proven to be invariant under the following set of transformations :

$$\delta x^m = 0 \tag{13}$$

$$\delta\theta = \frac{\lambda}{2}\, i\tau_2\theta \tag{14}$$

$$\delta F_{\mu\nu} = \lambda K_{\mu\nu} \tag{15}$$

$$\delta K_{\mu\nu} = -\lambda F_{\mu\nu} \tag{16}$$

where $K_{\mu\nu} = -\frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\tilde{K}^{\rho\sigma}$ [4] and

$$\tilde{K}^{\rho\sigma} = \frac{1}{\sqrt{-\det\mathcal{G}}}\frac{\partial\mathcal{L}_{D3}}{\partial F_{\rho\sigma}}\,. \tag{17}$$

[3]We are using the same conventions for the Dirac matrices and forms as those given in [14].

[4]$\varepsilon_{\mu\nu\rho\sigma}$ denotes the covariantly constant antisymmetric tensor with indices raised and lowered by $\mathcal{G}_{\mu\nu}$.

One can check that

$$\begin{aligned} K_{\mu\nu} = &-\frac{\sqrt{-\det\mathcal{G}}}{\sqrt{-\det(\mathcal{G}+\mathcal{F})}}\left(\tilde{\mathcal{F}}_{\mu\nu} + \mathcal{T}\mathcal{F}_{\mu\nu}\right) \\ &+ C_{(2)\mu\nu}\,, \end{aligned} \tag{18}$$

where $\tilde{\mathcal{F}}$ stands for the Hodge dual of the two form $\mathcal{F}$ and $\mathcal{T} = \frac{1}{4}\mathcal{F}_{\mu\nu}\tilde{\mathcal{F}}^{\mu\nu}$.

Some comments are in order at this point. First of all, since it is the gauge invariant field strength $F_{\mu\nu}$ that transforms linearly (15), it is indeed true that the gauge potential V transforms non-locally as stressed in the introduction. In fact such a transformation was written down explicitly in the bosonic case in [16]. The only matter fields that transform are the fermionic scalars θ^i whose infinitesimal transformation coincides with the infinitesimal transformations of the corresponding supersymmetry generators Q^i used in the algebraic analysis.

To get a more physical intuition of the previous transformations and to further relate them with the algebraic discussion, we shall evaluate them on a well-known BPS configuration : *dyons*. These are $\nu = \frac{1}{4}$ solitons representing (p,q) strings ending on the D3-brane. They correspond to the array

$$\begin{array}{llllllllll} D3: & 1 & 2 & 3 & \_ & \_ & \_ & \_ & \_ & \_ \\ F1: & \_ & \_ & \_ & 4 & \_ & \_ & \_ & \_ & \_ \\ D1: & \_ & \_ & \_ & 4 & \_ & \_ & \_ & \_ & \_ \ . \end{array}$$

As all BPS configurations, they are characterized by some BPS equations

$$E^a = F_{0a} = \cos\alpha\,\partial_a y \tag{19}$$

$$B^a = \frac{1}{2}\epsilon^{abc}F_{bc} = \sin\alpha\,\delta^{ab}\partial_b y \quad a,b,c = 1,2,3 \tag{20}$$

where y stands for the transverse excited scalar field (along the 4th spacetime direction, as indicated in the previous array), and some supersymmetry projection conditions

$$\Gamma_{0123}\, i\tau_2\epsilon = \epsilon \tag{21}$$

$$\Gamma_{0y}\,(\cos\alpha\,\tau_3 + \sin\alpha\,\tau_1)\,\epsilon = \epsilon\,. \tag{22}$$

Equation (21) tells us our solution is describing a D3-brane along directions 123, whereas (22) describes a non-threshold bound state of strings (τ_3 factor) and D-strings (τ_1 factor). Such an interpretation is further confirmed by computing its energy. The latter is given by

$$E_{BPS} = E_{D3} + \sqrt{\left(Y_4^{(3)}\right)^2 + \left(Y_4^{(1)}\right)^2}\,, \tag{23}$$

where E_{D3} corresponds to the vacuum energy of an infinite planar D3-brane along 123 directions, whereas the second factor corresponds to the energy of a non-threshold bound state of strings and D-strings. The computation of (23) is entirely field theoretical, and in particular, we can express $Y_4^{(a)}$ in terms of the worldspace integrals

$$Y_4^{(3)} = \int_{D3} \vec{E}\cdot\vec{\nabla}y \quad , \quad Y_4^{(1)} = \int_{D3} \vec{B}\cdot\vec{\nabla}y\,. \qquad (24)$$

Equation (23) matches the pure algebraic analysis result. This would have been derived by solving the eigenvalue problem [9]

$$[\Gamma^4\left(\tau_1\,Y_4^{(1)} + \tau_3\,Y_4^{(3)}\right) + \Gamma^{123} i\tau_2\,Y_{123}]|\alpha >= E_{BPS}|\alpha > \ .$$

Let us evaluate transformations (15)-(16) for this configuration. From the on-shell equalities $\sqrt{-\det\mathcal{G}}\tilde{F}^{0a} = B^a$ and $\sqrt{-\det\mathcal{G}}\tilde{F}^{ab} = \epsilon^{abc}F_{0c}$, one derives

$$\begin{aligned}\mathcal{T} &= \frac{F_{0a}B^a}{\sqrt{-\det\mathcal{G}}} \\ \tilde{F}_{0a} &= -\tilde{F}^{0b}\left(\delta_{ba} + \partial_b y\partial_a y\right)\,, \end{aligned} \qquad (25)$$

which allow us to find

$$\begin{aligned}K_{0a} &= -B^a \\ K_{ab} &= \epsilon_{abc}F_{0c}\,, \end{aligned} \qquad (26)$$

where we have used that

$$\sqrt{-\det\left(\mathcal{G}+\mathcal{F}\right)} = 1 + \sin^2\alpha\delta^{ab}\partial_a y\partial_b y\,.$$

The latter lead to the well-known $SO(2)$ infinitesimal transformations

$$\begin{aligned}\delta E^a &= -\lambda B^a \\ \delta B^a &= \lambda E^a\,, \end{aligned} \qquad (27)$$

whose finite form is

$$\begin{aligned}E'^a &= \cos\lambda\, E^a - \sin\lambda\, B^a = \cos(\alpha+\lambda)\partial_a y \\ B'^a &= \sin\lambda\, E^a + \cos\lambda\, B^a = \sin(\alpha+\lambda)\partial_a y\,. \end{aligned} \qquad (28)$$

These transformations show that indeed we are rotating the field theory realization of the 'central charges' appearing in the supersymmetry algebra (24), so that we have indeed realized the forementioned automorphism as a non-local symmetry on the D3-brane effective action. Furthermore, as stressed in [9], by fine tuning the parameter of the transformation (λ), one can set one of the charges of the non-threshold bound state to zero, the above computation being a particular example of such behaviour [5]. We would like to close this section by pointing out that the SO(2) rotation among the electric and magnetic fields could have been derived by requiring kappa symmetry covariance. This is that the solution to $\Gamma_\kappa\epsilon = \epsilon$ is mapped to the corresponding solution to $\Gamma'_{\kappa'}\epsilon' = \epsilon'$, where $\Gamma'_{\kappa'}$ depends on the transformed fields and $\epsilon' = U_2(\alpha)\epsilon$.

[5]It is when one goes to the quantum theory, that the $\mathrm{SL}(2,\mathbb{R})$ automorphism group becomes an $\mathrm{SL}(2,\mathbb{Z})$ group, in order to be consistent with charge quantization.

III. T-DUALITY AND D2-BRANES PICTURE

In the previous section, we discussed some automorphisms of type IIB SuperPoincaré algebra and in particular, the way its $SO(2)$ subgroup was realized non-locally on the dynamical fields describing the D3-brane effective action. It is natural to wonder about this symmetry structure in type IIA, both algebraically and from the D2-brane field theory perspective. We leave the M-theory interpretation for the next section.

It is already known the relation among type IIA and IIB SuperPoincaré algebras. If a T-duality is performed along a spacelike direction s, one may choose the supercharges to be related as follows

$$Q^+ = Q^2 \quad , \quad Q^- = \Gamma_s Q^1\,, \qquad (29)$$

where $Q^\pm$ are the type IIA supercharges. In this way, the previous $U_2(\alpha)$ automorphism can be rewritten as $U_s = e^{\alpha/2\,\Gamma_s\Gamma_{11}}$, which indeed belongs to SO(32), the subgroup of type IIA automorphisms preserving energy. The latter statement can be straightforwardly derived from the M-algebra analysis done in [9]. Notice that Γ_{11} is the ten dimensional chirality operator, so that $U_s(\alpha)$ can not be interpreted as an spacetime rotation. From the type IIA superalgebra perspective, it has to do with the freedom one has to make the choice (29) when relating both superalgebras under T-duality. As discussed before, such automorphisms will "rotate" several doublets of charges appearing in type IIA, while keeping some others invariant. In particular, charges $\mathcal{Z}_{sm}$ and $\mathcal{Z}_m$ corresponding to D2-branes and fundamental strings will form an SO(2) doublet under $U_s(\alpha)$ transformations. We refer the reader to [9] for such a discussion.

Once we apply a T-duality transformation, we loose spacetime covariance, but it is clear that one could have performed a T-duality along a different spacelike direction s', so that it should be expected not just a single transformation but a set of them, $U_m = e^{\alpha_m/2\Gamma_m\Gamma_{11}}$ $m = 1,\ldots,9$ to be relevant in the T-dual description. This will be confirmed in the field theory analysis.

Let us move to the field theory perspective. In [17,18], the way longitudinal T-duality is realized on D-brane effective actions was studied, not only at the level of the action functional but also on its symmetry structure. The latter will be particularly useful for us in order to derive the symmetry structure inherited by the D2-brane from its T-dual D3-brane. Since no bosonic scalar field (x^m) transforms under $U_2(\alpha)$ (see equation (13)), there will be no compensating diffeomorphism transformation coming from the partial gauge fixing involved in the world volume realization of T-duality ($x^s = \rho$) [6]. Thus, we can directly

[6]Spacetime coordinates $\{x^m\}$ are splitted into $\{x^{\hat{m}}, x^s\}$

study the double dimensional reduction of the transformation laws (14)-(16). Let us start from the fermionic sector. As shown in [18], the relation among type IIA and type IIB fermions is given by

$$\theta_+ = \theta_2 \quad , \quad \theta_- = \Gamma_s \theta_1 \,. \tag{30}$$

Just as for the supersymmetry generators, it is possible to rewrite (14) as

$$\delta\theta = \frac{\lambda}{2}\Gamma_s\Gamma_{11}\theta \,, \tag{31}$$

where, as usual, $\theta = \theta_+ + \theta_-$, the subindex indicating its ten dimensional chirality.

Since under T-duality, one of the gauge field components (V_ρ) becomes a world volume scalar in the T-dual theory ($\tilde{x}^s$), it must be expected to get non-local transformations not just for the reduced gauge field components ($V_{\hat{\mu}}$) but also for $\tilde{x}^s$. Double dimensional reduction of transformations (15) and (16) gives

$$\delta K_{\hat{\mu}\hat{\nu}} = -\lambda F_{\hat{\mu}\hat{\nu}} \quad , \quad \delta F_{\hat{\mu}\hat{\nu}} = \lambda K_{\hat{\mu}\hat{\nu}} \tag{32}$$

$$\delta K_{\hat{\mu}\rho} = -\lambda \partial_{\hat{\mu}}\tilde{x}^s \quad , \quad \delta \partial_{\hat{\mu}}\tilde{x}^s = \lambda K_{\hat{\mu}\rho} \tag{33}$$

whereas the remaining scalars do still remain invariant ($\delta x^{\hat{m}} = 0$). In the latter expressions, $K_{\hat{\mu}\hat{\nu}}$ and $K_{\hat{\mu}\rho}$ are given by

$$\begin{aligned} K_{\hat{\mu}\hat{\nu}} = & -\frac{1}{\sqrt{-\det(\mathcal{G}+\mathcal{F})}}\Big[\mathcal{G}_{\hat{\mu}\hat{\alpha}}\mathcal{G}_{\hat{\nu}\hat{\beta}}\epsilon^{\hat{\alpha}\hat{\beta}\hat{\delta}}\Pi^s_{\hat{\delta}} \\ & + \left(\frac{1}{2}\epsilon^{\hat{\alpha}\hat{\beta}\hat{\delta}}\mathcal{F}_{\hat{\alpha}\hat{\beta}}\Pi^s_{\hat{\delta}}\right)\mathcal{F}_{\hat{\mu}\hat{\nu}}\Big] \\ & + \bar{\theta}\hat{\Pi\!\!\!/}_{\hat{\mu}}\Gamma_s\Gamma_{11}\partial_{\hat{\nu}}\theta - (\hat{\mu}\leftrightarrow\hat{\nu}) \,, \end{aligned} \tag{34}$$

$$\begin{aligned} K_{\hat{\mu}\rho} = & -\frac{1}{\sqrt{-\det(\mathcal{G}+\mathcal{F})}}\left[\frac{1}{2}\mathcal{G}_{\hat{\mu}\hat{\alpha}}\epsilon^{\hat{\alpha}\hat{\beta}\hat{\delta}}\mathcal{F}_{\hat{\beta}\hat{\delta}}\right] \\ & + \bar{\theta}\Gamma_{11}\partial_{\hat{\mu}}\theta \,. \end{aligned} \tag{35}$$

It must be understood that all fields appearing in (34) and (35) are type IIA fields, furthermore, the $(\hat{\mu}\leftrightarrow\hat{\nu})$ prescription just applies to the third line in (34).

Notice that transformation (34) is manifestly non-covariant, since it depends on the direction along which we perfom T-duality, whereas (35) is totally covariant. In order to check whether the T-dual D2-brane action has more non-local symmetries than the ones described before, we shall compute the commutator of a ten dimensional Lorentz transformation (ω^{mn}) and one of our new symmetry transformations (λ) :

$$[\delta\,,\tilde{\delta}]\theta = \lambda\omega^{sp}\Gamma_p\Gamma_{11}\theta = \tilde{\lambda}^p\Gamma_p\Gamma_{11}\theta \,. \tag{36}$$

whereas world volume ones according to $\{\sigma^\mu\} = \{\sigma^{\hat{\mu}}, \rho\}$.

Due to the antisymmetry of the Lorentz parameter ω^{sp}, p is definitely different from s. This shows that our three dimensional field theory has a larger set of non-local transformations [7] which can be obtained just by making covariant the previous ones,

$$\delta\theta = \tfrac{\lambda}{2}\Gamma_m\Gamma_{11}\theta \,, \tag{37}$$

$$\delta K^m_{\hat{\mu}\hat{\nu}} = -\lambda^m F_{\hat{\mu}\hat{\nu}} \quad , \quad \delta F_{\hat{\mu}\hat{\nu}} = \lambda^m K^m_{\hat{\mu}\hat{\nu}} \tag{38}$$

$$\delta K_{\hat{\mu}\rho} = -\lambda^m \partial_{\hat{\mu}}x^m \quad , \quad \delta\partial_{\hat{\mu}}x^m = \lambda^m K_{\hat{\mu}\rho} \tag{39}$$

where

$$\begin{aligned} K^m_{\hat{\mu}\hat{\nu}} = & -\frac{1}{\sqrt{-\det(\mathcal{G}+\mathcal{F})}}\Big[\mathcal{G}_{\hat{\mu}\hat{\alpha}}\mathcal{G}_{\hat{\nu}\hat{\beta}}\epsilon^{\hat{\alpha}\hat{\beta}\hat{\delta}}\Pi^m_{\hat{\delta}} \\ & + \left(\frac{1}{2}\epsilon^{\hat{\alpha}\hat{\beta}\hat{\delta}}\mathcal{F}_{\hat{\alpha}\hat{\beta}}\Pi^m_{\hat{\delta}}\right)\mathcal{F}_{\hat{\mu}\hat{\nu}}\Big] \\ & + \bar{\theta}\hat{\Pi\!\!\!/}_{\hat{\mu}}\Gamma^m\Gamma_{11}\partial_{\hat{\nu}}\theta - (\hat{\mu}\leftrightarrow\hat{\nu}) \end{aligned} \tag{40}$$

We would like to stress that such 'enhancement' of symmetry is typical of T-duality and it is certainly not constrained to the particular construction used here.

Just as for the D3-brane case, we shall analyze the behaviour of some particular BPS configuration under these new transformations. We shall T-dualize the previous dyonic configuration along the direction 3. The T-dual array is given by

$$\begin{array}{llllllllllll} D2: & 1 & 2 & \_ & \_ & \_ & \_ & \_ & \_ & \_ & \_ & \\ F1: & \_ & \_ & \_ & 4 & \_ & \_ & \_ & \_ & \_ & \_ & \\ D2: & \_ & \_ & 3 & 4 & \_ & \_ & \_ & \_ & \_ & \_ & . \end{array}$$

By the analysis done in [18], it is known that its BPS equations are described by the double dimensional reduction of equations (19) and (20)

$$F_{0\hat{a}} = \cos\alpha\,\partial_{\hat{a}}y \tag{41}$$

$$\epsilon^{\hat{a}\hat{b}}\partial_{\hat{b}}\tilde{x}^3 = \sin\alpha\,\delta^{\hat{a}\hat{b}}\partial_{\hat{b}}y \quad \hat{a},\hat{b} = 1,2 \tag{42}$$

$$F_{12} = 0 \,. \tag{43}$$

The third equation states that there are no D0-branes being described by our configuration as can be further confirmed by looking at the supersymmetry projection conditions

$$\Gamma_{012}\epsilon = \epsilon \tag{44}$$

$$(\cos\alpha\,\Gamma_{0y}\Gamma_{11} + \sin\alpha\,\Gamma_{03y})\,\epsilon = \epsilon \tag{45}$$

which are obtained from (21)-(22) by direct application of the fermionic rules (30). Notice that when $\alpha = 0$, we recover the usual BIon describing a fundamental string ending on the D2-brane, whereas for $\alpha = \frac{\pi}{2}$, we recover

[7] Similar arguments can be applied to the remaining bosonic scalar fields on the brane.

the Cauchy-Riemann equations describing the intersection of two D2-branes at a point, $D2 \perp D2(0)$. Both configurations are related to each other by application of transformations (38) and (39). Computing them when (41)-(43) are satisfied or just by double dimensionally reducing equations (27) we get

$$\begin{aligned} \delta\vec{E} &= -\lambda \star \nabla\tilde{x}^3 \\ \delta\left(\star\nabla\tilde{x}^3\right) &= \lambda\vec{E}\,, \end{aligned} \tag{46}$$

where we are using the standard two dimensional calculus notation, that is, $\vec{\nabla} = (\partial_1\,,\partial_2)$ and $\star\vec{\nabla} = (\partial_2\,,-\partial_1)$. Its finite transformation is

$$\begin{aligned} \vec{E}' &= \cos\lambda\,\vec{E} - \sin\lambda\,\star\vec{\nabla}\tilde{x}^3 \\ &= \cos(\alpha+\lambda)\,\vec{\nabla}y \\ \star\vec{\nabla}\tilde{x}'^3 &= \sin\lambda\,\vec{E} + \cos\lambda\,\star\vec{\nabla}\tilde{x}^3 \\ &= \sin(\alpha+\lambda)\,\vec{\nabla}y \end{aligned} \tag{47}$$

Thus, as expected, by fine tuning the global parameter λ, we interpolate between BIon configurations and $D2 \perp D2(0)$ intersections.

The SO(2) rotation described by (46) fits with the supersymmetry algebra picture. In this case, the charge carried by the fundamental string is given by the worldspace integral

$$\mathcal{Z}_y = \int_{D2} \vec{E}\cdot\vec{\nabla}y\,, \tag{48}$$

whereas the charge carried by the second D2-brane admits the field theory realization

$$\mathcal{Z}_{3y} = \int_{D2} \star\vec{\nabla}x^3\cdot\vec{\nabla}y\,. \tag{49}$$

Thus we see that $\mathcal{Z}_y$, $\mathcal{Z}_{3y}$ are indeed rotated under (46) transformations, as the pure algebraic digression was suggesting to us.

IV. M-THEORY INTERPRETATION

It is natural to wonder about the M-theory interpretation of any symmetry structure found in type IIA theory. In the following, we shall reformulate all previous results from an eleven dimensional perspective, both algebraically and from a field theory scenario. Due to the relation among $\mathcal{N}=1$ $D=11$ SuperPoincaré algebra and $\mathcal{N}=2$ $D=10$ type IIA SuperPoincaré algebra, the latter being the dimensional reduction of the former, it is straightforward to reinterpret the previous $SO(32)$ automorphisms as $SO(10)$ rotations, which we shall denote as $U_m(\alpha) = e^{\alpha_m/2\Gamma_m\Gamma_\sharp}$ [8]. Due to its rotational character, preservation of energy is guaranteed. This picture also agrees with the well-known fact that an S-duality transformation ($\alpha = \frac{\pi}{2}$) in type IIB is seen as a rotation interchanging the two independent cycles in the two torus needed to relate M-theory with type IIB string theory [14].

The advantage of the M-theory formulation is that the previous non-local transformations will be linearly realized on the M2-brane effective action. It is actually very simple to match both results. Since the D2-brane effective action is related to the M2-brane one by a world volume dualization [13], the linearly realized rotation ($\omega^{m\sharp}$) will induce a linear transformation on the gauge invariant quantity $\mathcal{F}$, but a non-local one on the abelian $U(1)$ gauge field, as discussed previously.

Let us look into this connection more closely. Consider the three dimensional M2-brane effective action propagating in SuperPoincaré [19]. The latter is invariant under the global $SO(10)$ rotations

$$\begin{aligned} \delta x^m &= \omega^{my}\,y \\ \delta y &= -\omega^{my}\,x_m \\ \delta\theta &= \frac{1}{2}\omega^{my}\,\Gamma_m\Gamma_\sharp\,\theta\,, \end{aligned} \tag{50}$$

where $x_m = \eta_{mn}x^n$ and we have already splitted the eleven dimensional bosonic scalar fields into $\{x^m, y\}$. The basic equation relating the scalar field y with its three dimensional dual V is given by

$$\partial_\mu y - \bar{\theta}\Gamma_\sharp\partial_\mu\theta = \frac{1}{2}\frac{v}{det\,\mathcal{G}^{(10)}}\mathcal{G}^{(10)}_{\mu\nu}\epsilon^{\nu\alpha\beta}\mathcal{F}_{\alpha\beta}\,, \tag{51}$$

where v is an auxiliary scalar density whose value can be computed by solving its classical equation of motion

$$v = \sqrt{-\det\mathcal{G}^{(11)}} = \frac{-\det\mathcal{G}^{(10)}}{\sqrt{-\det\left(\mathcal{G}^{(10)}+\mathcal{F}\right)}}\,. \tag{52}$$

Comparing with the objects appearing in the D2-brane transformations, we realize that equation (51) is equivalent to

$$\partial_\mu y = K_{\mu\rho}\,. \tag{53}$$

From equation (53), we recover the set of transformations for the dynamical fields on the D2-brane,

$$\delta\partial_\mu y = -\omega^{my}\partial_\mu x_m \Leftrightarrow \delta K_{\mu\rho} = -\omega^{my}\partial_\mu x_m \tag{54}$$

$$\delta\partial_\mu x^m = \omega^{my}\partial_\mu y \Leftrightarrow \delta\partial_\mu x^m = \omega^{my}K_{\mu\rho}\quad, \tag{55}$$

whereas the fermionic transformations are trivially identified since the eleven dimensional Majorana spinors θ are splitted into $\theta_1+\theta_2$, the two different chiral Majorana-Weyl spinors in type IIA. Thus the linear transformations (50) are mapped, through the world volume dualization (51), to non-local transformations on the D2-brane action, by identifying $\omega^{my} = \lambda^m$. Notice that this eleven

[8] We use the symbol $\sharp$ to refer to the eleven dimensional direction, as it is usually done in the literature.

dimensional interpretation again agrees with the symmetry enhancement found in the D2-brane when looking at it from its T-dual D3-brane.

To finish up the M-theory discussion, we shall comment on the uplifted BPS configuration corresponding to the previous section. This is described by the array

$$\begin{array}{l} M2:\ 1\ 2\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_ \\ M2:\ \_\ \_\ \_\ 4\ 5\ \_\ \_\ \_\ \_\ \_ \\ M2:\ \_\ \_\ 3\ 4\ \_\ \_\ \_\ \_\ \_\ \_\ . \end{array}$$

We shall set the static gauge $x^\mu = \sigma^\mu$ $\mu = 0,1,2$ to describe the location of the world volume brane and excite three transverse scalars x^i $i = 3,4,5$. We shall find the BPS equations for this configuration by solving the kappa symmetry preserving condition,

$$\frac{1}{3!}\epsilon^{\mu\nu\rho}\partial_\mu x^m \partial_\nu x^n \partial_\rho x^p \Gamma_{mnp}\epsilon = \sqrt{-\det \mathcal{G}}\,\epsilon\,. \tag{56}$$

Just from inspection of the latter array, three single brane projectors will be involved in the solution $\{\Gamma_{012}, \Gamma_{045}, \Gamma_{034}\}$. Following the analysis presented in [9], the supersymmetry projection conditions must be

$$\Gamma_{012}\epsilon = \epsilon \tag{57}$$

$$(\cos\alpha\,\Gamma_{045} + \sin\alpha\,\Gamma_{034})\,\epsilon = \epsilon\,. \tag{58}$$

The left hand side of equation (56) equals

$$\left(\Gamma_{012} + \epsilon^{0ab}\partial_b x^i \Gamma_{0ai} + \frac{1}{2}\epsilon^{0ab}\partial_a x^i \partial_b x^j \Gamma_{0ij}\right)\epsilon$$

where $a,b = 1,2$. Requiring $\epsilon^{0ab}\partial_b x^i \Gamma_{0ai}\epsilon$ to vanish and using the identities

$$\begin{aligned} \Gamma_{014}\epsilon &= -\left(\cos\alpha\,\Gamma_{025} + \sin\alpha\,\Gamma_{023}\right)\epsilon \\ \Gamma_{024}\epsilon &= \left(\cos\alpha\,\Gamma_{015} + \sin\alpha\,\Gamma_{013}\right)\epsilon\,, \end{aligned} \tag{59}$$

derived from (57) and (58), we get the set of BPS equations

$$\cos\alpha\,\vec{\nabla}x^4 = \star\vec{\nabla}x^5 \tag{60}$$

$$\sin\alpha\,\vec{\nabla}x^4 = \star\vec{\nabla}x^3\,. \tag{61}$$

Notice that they interpolate among $M2 \perp M2(0)$ configurations in definite directions for $\alpha = 0, \frac{\pi}{2}$. The remaining non-trivial term $\frac{1}{2}\epsilon^{0ab}\partial_a x^i \partial_b x^j \Gamma_{0ij}$ splits into

$$\frac{1}{2}\epsilon^{0ab}\partial_a x^{\hat{\imath}} \partial_b x^{\hat{\jmath}} \Gamma_{0\hat{\imath}\hat{\jmath}} + \frac{1}{2}\epsilon^{0ab}\partial_a x^5 \partial_b x^3 \Gamma_{053}\,. \tag{62}$$

The last one is identically zero due to (60) and (61), whereas the first one equals

$$(x^4)^2\,(\cos\alpha\,\Gamma_{045} + \sin\alpha\,\Gamma_{034})\,\epsilon = (x^4)^2\epsilon \tag{63}$$

by (60)-(61) and (58). Thus the left hand side equals $(1 + (x^4)^2)\epsilon$.

To check the matching with the right hand side, we must compute the world volume induced metric when (60) and (61) are satisfied. This is given by

$$\mathcal{G}_{00} = -1 \quad , \quad \mathcal{G}_{0a} = \mathcal{G}_{12} = 0 \tag{64}$$

$$\mathcal{G}_{11} = 1 + \textstyle\sum_i (\partial_1 x^i)^2 = 1 + (x^4)^2 \tag{65}$$

$$\mathcal{G}_{22} = 1 + \textstyle\sum_i (\partial_2 x^i)^2 = 1 + (x^4)^2 \tag{66}$$

and indeed shows that $\sqrt{-\det\mathcal{G}}\,\epsilon = (1 + (x^4)^2)\epsilon$, matching our previous computation.

At this point, one can check the existence of an $SO(10)$ rotation relating the latter BPS configuration with

$$\begin{array}{l} M2:\ 1\ 2\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_ \\ M2:\ \_\ \_\ \_\ 4\ 5\ \_\ \_\ \_\ \_\ \_\ . \end{array}$$

Following our general discussion, such a rotation must be $U = e^{\beta\Gamma_{53}/2}$. Indeed, this rotation in the 35-plane transforms the BPS equations (60)-(61) into

$$\star\,\vec{\nabla}x'^5 = \cos(\alpha+\beta)\,\vec{\nabla}x^4 \tag{67}$$

$$\star\vec{\nabla}x'^3 = \sin(\alpha+\beta)\,\vec{\nabla}x^4 \tag{68}$$

which show that by setting $\beta = -\alpha$, x'^3 becomes constant, and there is thus no longer an excited scalar in that direction. The second supersymmetry condition (58) is also conveniently mapped into $\Gamma_{045}\epsilon' = \epsilon'$, confirming our previous interpretation.

Rotations in the 35-plane indeed rotate the corresponding M2-brane charges in the supersymmetry algebra, since they are given by

$$\begin{aligned} \mathcal{Z}_{45} &= \int_{M2} \star\vec{\nabla}x^5 \cdot \vec{\nabla}x^4\,, \\ \mathcal{Z}_{34} &= \int_{M2} \star\vec{\nabla}x^3 \cdot \vec{\nabla}x^4\,. \end{aligned} \tag{69}$$

Once we have understood the M-theory configuration, it is straightforward to recover all previous D2-brane BPS equations (19)- (20) and supersymmetry conditions (21)-(22) by explicitly using the relation (53) on-shell. This finishes the consistency check of the exposed non-local transformations into the web of M/string theory dualities.

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